A Method for Repairing Triangulations

Alexander Emelyanov, Vaclav Skala
Department of Computer Science
University of West Bohemia, Plzen, Czech Republic
{aiem, skala}@kiv.zcu.cz

Abstract

In practice of surface reconstruction it often happens that a used algorithm is not capable to reconstruct a CAD-model completely. In this work a method to repair defects of a CAD-model is offered. For surface reconstruction inside damaged regions this method uses force lines of a specially constructed tensor field. This method can be considered as a post-processing step for any surface reconstruction algorithm.

Keywords: surface reconstruction, repairing triangulations.

1. FORMALIZATION OF THE PROBLEM

Definition 1. Let's consider an unsuccessful result of work of some surface reconstruction algorithm. Let we have a particularly reconstructed surface, and all regions of this surface (let's denote the aggregate of them A) are topologically equivalent to the corresponding regions of the original surface. Let's assume there are no invalid edges and triangles in the considered result. It means that each point of the input cloud of points is included in A or doesn't have any triangles and edges (let's call such point a *free point*). Let's call such result an *incomplete normalized CAD-model* (*ICADM*). Let's denote the aggregate of regions supplementing A up to a topologically correct CAD-model $\overline{A}$.

In the given work a method for obtaining a CAD-model of a given object from an ICADM is described. This method is designed with the assumption, that information contained in the given ICADM is sufficient for obtaining a result with practical value. In general, an unsuccessful result of work of a surface reconstruction algorithm isn't an ICADM. Reduction of such result to ICADM usually requires application of a corresponding method of filtering. One of these methods is described in [4]. In general, A may consist of one coherent surface region or several ones isolated from each other (let's call such isolated regions *islands*). As a rule, there are holes on A. Thus A has a set of closed boundaries. This set consists at least of one element. An element of this set can be the boundary of a hole or the boundary of an island.

Definition 2. Let there be a hole. Let's call this hole a *simple hole* if a proper surface inside it can be reconstructed by an existing fast and simple surface reconstruction method (let's call such method a *"darning" method*), and a *complex hole* otherwise.

Naturally, that the same hole can be simple concerning a given "darning" method and can be complex concerning another one. For example the hole in figure 1 is most likely simple concerning a spline-based method and is complex concerning a projection-based method. For the further statement only the fact, that reconstruction of a proper surface inside a simple hole is not a problem, is important.

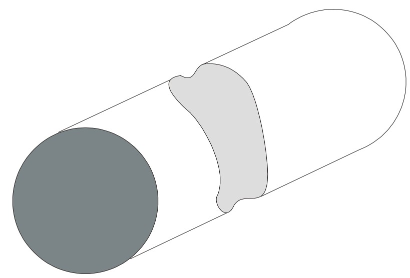


Figure 1

When considering the problem of obtaining a correct CAD-model from a given ICADM it is necessary to take into account, that reconstruction of $\overline{A}$ can be made only on the basis of analysis of behavior of A and distribution of free points. These points typically have the following properties:

- density and uniformity of distribution are sufficiently smaller than the corresponding indices of points of A;
- inaccuracy of determination of the coordinates is greater than this index of points of A.

Thus, usually we don't have necessary information to provide a complicated behavior of reconstructed $\overline{A}$. And usually we can't reconstruct $\overline{A}$ with the same precision like A. In general, adequacy of reconstruction of $\overline{A}$ has a likelihood character. Taking into account these considerations let's work out the problem with the assumption that behavior of $\overline{A}$ isn't too complicated. In general, probability of adequate reconstruction of $\overline{A}$ is higher in regions where A has a simple behavior.

Definition 3. Each ICADM can be related to one of the following classes:

ICM1: A is represented by one coherent region, that has only simple holes;

ICM2: A is represented by one coherent region too, but there are both simple and complex holes inside;

ICM3: A is represented by the aggregate of several islands, which have possible holes of both types.

In accordance with definition 2 obtaining a correct CAD-model from a given ICADM of class ICM1 is not a problem. Thus, our problem is to find a method of correct processing ICADMs of classes ICM2 and ICM3.

2. DEVELOPMENT OF A NEW SOLUTION MOTIVATION

Among existing surface reconstruction methods warping methods based on physical modeling [1, 2, 3] have necessary features to solve the above-formulated problem. This technique means, that there is an initial membrane (mesh) that is topologically equivalent to the original surface of a given object (in most cases it is the topology of a closed sphere). Elements of the membrane interact both with each other and with sampled points by rules defined by a given physical model. As a result of this interaction the membrane takes deformation to approximate the original surface of the object. A system of material points connected by springs is usually used as the interaction model. Material points corresponding to sampled points are considered fixed. Such model can be expressed as a system of linear differential equations of degree 2. This system is solved iteratively. In general, application of the warping-based methods to solve the considered problem has the following advantages and disadvantages.

Advantages:

- If a modeled membrane at the beginning of the process is topologically equivalent to the original object surface, then topologically correctness of reconstructed $\overline{A}$ is provided.
- $\overline{A}$ is represented by a region (regions) of the surface of minimal potential energy of the modeled membrane. It is the most probable approximation of the original surface, and in many cases the condition of smoothness is satisfied as well.
- ICADMs of all the three classes can be processed by the same method.

Disadvantages:

- Topological correctness of $\overline{A}$ is provided by the fact that during the modeling the membrane is considered entirely. Of course, at the final step only regions of the membrane corresponding to $\overline{A}$ can be modeled, but cost of elimination form the modeling regions of the membrane corresponding to A masks optimization effect. Therefore, cost of obtaining a CAD-model from a given ICADM is not essentially smaller than cost of obtaining the same result from the corresponding cloud of points.
- In general, the process of modeling is costly enough. To reduce this cost many existing methods use the warping technique only at the final step to obtain a total shape of the membrane. An initial shape of the membrane approximating the original object surface is obtained by using a different algorithm that is not so costly. But for ICADMs of classes ICM2 and ICM3 (especially) such algorithm often can't make an initial shape of the membrane with the correct topology.

Thus, for solution of our problem it would be good to find a method with the following properties:

- cost depending only on the number of points included in $\overline{A}$ (they are points on boundaries of A and free points);
- a capability to determine the correct topology of $\overline{A}$ with cost smaller than the cost of the modeling membrane and with robustness greater than the robustness of existing fast surface reconstruction algorithms;
- a capability to provide behavior of $\overline{A}$ in accordance with the surface of minimal potential energy of a membrane that is correctly connected to boundaries of A.

3. BASIC CONCEPTS

3.1 Concept of bridges

Let's note that to reduce of an ICADM of class ICM3 to an ICADM of class ICM2 and then to an ICADM of class ICM1 it is enough to make reconstruction of $\overline{A}$ only in properly chosen local regions (figure 2).

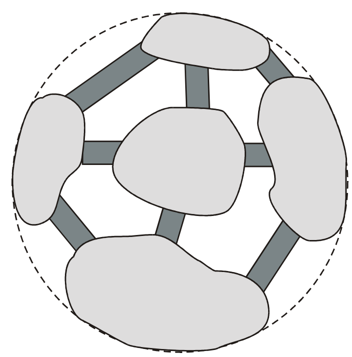


Figure 2

Definition 4. The simplest case of such local region is a curve connecting two selected boundary points of A with definition of the external normal vector along the curve (figure 3). Let's call such curve a *bridge* and the boundary points (A and B in the figure) connected by this bridge its *supporting points* Because of the fact that at each point of a bridge the corresponding normal vector is defined we can interpret a bridge as an infinite narrow surface strip. In the further statement the term "*the surface of a bridge"* is used in the sense of this interpretation.

.

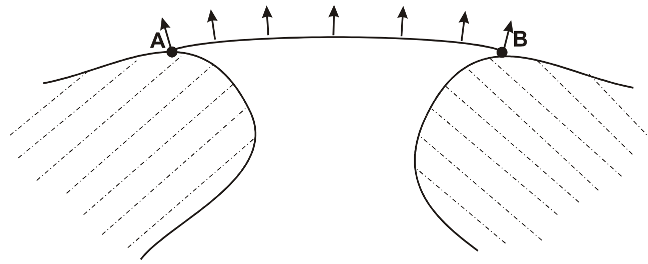


Figure 3

Definition 5. Let's define *quality* of a given bridge as estimated in some way probability of the bridge to be topologically correct.

3.2 Concept of an indicator field

To implement step ICM3→ICM2 for a given set of islands we need to determine a proper set of bridges and these bridges should have the quality as high as possible. The most natural way to solve this problem is consecutive consideration of each of the given islands (let's call a currently considered island the *base island*) and determination for it a set of the best quality bridges with other islands. The total set of bridges is determined on the base of union of such sets.

Let's assume that the boundaries of islands are sources of a special field (let's call it *indicator field*, H). For a given base island possible bridges connecting it with other islands are defined by force lines of this field (figure 4). These force lines come out of points on the boundary of the base island and go to the boundaries of other islands.

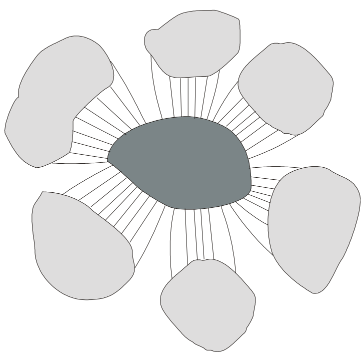


Figure 4

Let's assume that this field contains sufficient information to determine both: the trajectory of a force line as well as the external normal vector along this trajectory.

Let's assume that the average scalar value of the field tension along a given force line is the value of the quality of the bridge represented by this force line.

Let's note, that in accordance with the last assumption the quality of a bridge with a high enough probability is proportional to the scalar value of the tension at its origin point. Taking it into consideration, to determine the required bridges we can track the force lines outgoing from points on the boundary of the base island with the highest scalar values of the tension until we obtain a sufficient number of bridges with proper quality. A natural way to track a force line is application of an iterative method, when at each iteration step on the base of a current point of the force line we obtain the next point of this line by analyzing the field tension at the current point.

Realization of step ICM2→ICM1 can be based on the same principles in general. At this step decomposition of a given complex hole is made by recursive binary subdivision of the hole until each resulting hole is a simple hole. The subdivision is made by construction of a bridge inside the hole contour.

4. FORMALIZATION OF THE INDICATOR FIELD

4.1 Formalization of the quality of a simple bridge

4.1.1 Base conditions

Definition 6. Since each island is a part of a reconstructed surface of an object, we can determine the external and the internal sides of the given island. Let's call an island with determined sides an *oriented* island. For the island let's define the *positive direction* of movement along its boundary the counter-clockwise direction, if we look to the external side.

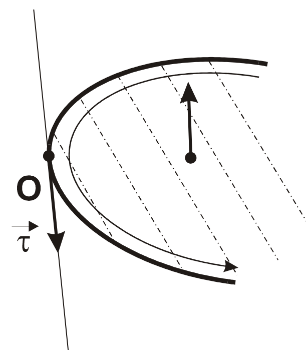


Figure 5

Definition 7. Let's consider an island, a point O on its boundary, and the tangent line to the boundary at this point (figure 5). Let's call the unit vector $\vec{\tau}$ on the tangent line with the origin at O directed in accordance with the positive direction of the boundary the *tangent vector* to the boundary at the given point.

Let's consider two points (let's denote them A and B) on the boundaries of given islands (let's denote them X and Y respectively). Let's consider the segment between A and B . Firstly let's define the two following vectors:

$$\vec{s}^{BA} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} \times \vec{\tau}^{A} \quad (1a)$$

$$\vec{s}^{AB} = \frac{\overrightarrow{BA}}{|\overrightarrow{BA}|} \times \vec{\tau}^{B} \quad (1b)$$

where $\vec{\tau}^{A}$ and $\vec{\tau}^{B}$ are the tangent vectors at the points A and B respectively.

Condition 1. It is obvious, that to provide smooth junction with the given islands any bridge with the trajectory represented by the given segment should have in the points A and B the external normal vectors with the same direction as the vectors $\vec{s}^{BA}$ and $\vec{s}^{AB}$ respectively have.

Condition 2. Let's define a necessary condition for the considered segment to be the topologically correct trajectory of a bridge with the supporting points A and B by the following formula:

$$q_T = \vec{s}^{\,BA}\,\vec{s}^{\,AB} > 0 \qquad (2)$$

This expression can be interpreted in the following way. If we go from a point on the external side of X to the segment, at point A we are on the segment and the external normal vector has the same direction as $\vec{s}^{\,BA}$ has. If we go along the segment to island Y keeping this external normal direction, we end up on: the external side of Y if $q_T > 0$ (and "easiness" of the transition is proportional to q_T); the internal side of Y if $q_T < 0$; the boundary of Y if $q_T = 0$. It is obvious, that only the first case corresponds to a topologically correct connection between the islands.

4.1.2 The basic task

Let we have a given topology of islands, one of them is chosen as the base island. Let's consider (figure 6): O is a point on the boundary of an island, that isn't the base island; $\vec{n}^{\,O}$ is the external normal vector to the island surface at point O; $\vec{\tau}^{\,O}$ is the tangent vector at this point; X is a point in the space between the given islands, $\vec{n}^{\,X}$ is a unit vector at this point.

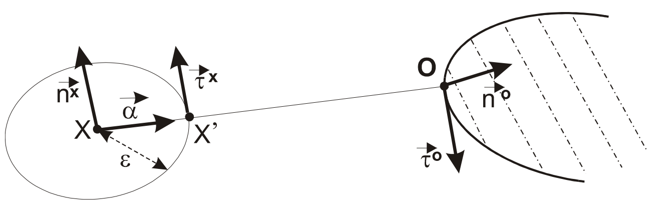


Figure 6

In the beginning let's assume that X and $\vec{n}^{\,X}$ define a round platform with an arbitrary small radius ε. This platform is perpendicular to $\vec{n}^{\,X}$ and has point X as the center. Let's consider this platform as an elementary island. Since ε is arbitrary small ($\lim_{\varepsilon \to 0} X' = X$) let's assume that any bridge connecting this elementary island with another island has as the supporting point on the elementary island point X.

Let's consider segment XO and define on its base the bridge XO by assumption that this bridge has the distribution of the external normal vector providing the simplest behavior of the bridge surface (with condition that condition 1 is fulfilled). Let's formalize the quality (let's denote it Q) of this bridge.

4.1.3 Formulation of other conditions

Firstly, let's note, that in accordance with condition 2 $Q \sim q_T$. Let's also take into account the following conditions that have influence on the quality:

Condition 3.1. The trajectory of the bridge should be as short as possible.

Condition 3.2. The trajectory of the bridge should lie in region that has as high concentration of free points as possible.

Condition 3.3. The trajectory of the bridge should lie in the plane defined by X and $\vec{n}^{\,X}$ as entirely as possible.

Condition 3.4. The trajectory of the bridge should interpolate behavior of the surface of the island containing point O in some neighborhood of this point as entirely as possible.

The conditions 3.1 and 3.2 are a consequence of the fact that the probability of correctness of the bridge increases if its trajectory lies near to sampled points. At the same time condition 3.1 is dominating because free points have lesser density and lesser trustworthiness of the coordinates. Also, for a bridge in general case, condition 3.1 expresses the fact that cost of construction of the bridge is proportional to its length. Condition 3.3 expresses the assumption (made in section 1) that behavior of $\overline{\mathrm{A}}$ isn't too complicated. Condition 3.4 expresses the fact that usually only A has a proper degree of trustworthiness.

Let's formalize the indices of fulfillment of these conditions by the considered bridge.

4.1.4 The index of fulfillment of condition 3.1

Let's consider that this index (let's denote it q_1) is defined by the function:

$$q_1 = q_1(L) \qquad (3)$$

which has the following properties:

$$q_1(L) \ge 0$$

$$\frac{dq_1}{dL} \le 0$$

$$q_1(0) = 1$$

$$\lim_{L \to \infty} q_1(L) = 0$$

4.1.5 The index of fulfillment of condition 3.2

To define this index let's assume that free points can compress the surrounding space (like sources of the gravitational field in the theory of relativity). As a result the length of a bridge passing in a neighborhood of free points shrinks. Let's assume as the value of the index the length of the bridge with taking into account such compression. To determine such length let's assume, that each free point is a source of an spherically symmetric scalar field $G = G(R)$, where R is the distance between the free point and a given space point. Let's assume that function $G(R)$ has the following properties:

$$G(R) \ge 0$$

$$\frac{dG}{dR} \le 0$$

$$G(0) = \frac{k_t}{1-k_t},$$

$$k_t \in (0,1)$$

$$\lim_{R\to\infty} G(R) = 0$$

where k_t is the "factor of trustworthiness" for the coordinates of the given free point.

Let's assume that an elementary segment with length dl located at a point A where the value of the field tension is G^A has the effective length $dl' = \frac{dl}{1+G^A}$. Therefore, in the case $R = 0$, if $k_t \to 0$ then $dl' \to dl$, and if $k_t \to 1$ then $dl' \to 0$.

Thus, the index (let's denote it q_2) can be expressed by this formula:

$$q_2 = \int_0^L \frac{dl}{1 + \sum_{i=1}^{m(\Omega)} G(|\vec{P_i} - \vec{Y}(l)|)} \qquad (4)$$

where Ω is the region around the considered bridge defined by the condition stating that only free points from this region have valuable influence on the length of the bridge; m is the total number of free points in Ω; P_i is the i-th free point of these free points; $\vec{Y} = \vec{Y}(l)$ is the parametric equation of the bridge; L is the length of the bridge.

4.1.6 The index of fulfillment of condition 3.3

We can formulate this index (let's denote it q_3) in the following way:

$$q_3 = |\sin(\vec{n}^X, \overrightarrow{XO})| = |\vec{n}^X \times \vec{d}| \qquad (5)$$

where

$$\vec{d} = \frac{\overrightarrow{XO}}{|\overrightarrow{XO}|} \qquad (6)$$

4.1.7 The index of fulfillment of condition 3.4

Let's define this index (let's denote it q_4) by the following formula:

$$q_4 = \frac{1 + \vec{s}^{XO} \vec{n}^O}{2} \qquad (7)$$

4.1.8 Derivation of the total quality index

To formalize the quality index of bridge XO we need to construct an expression based on the indices q_T, q_1, q_2, q_3, q_4 taking into account the logic of influence of each of these indices on the quality.

Firstly, it is naturally to unite the indices q_1 and q_2 (lets denote the united index q_{12}) by using q_2 as parameter L in the function defined by formula 3:

$$q_{12} = q_1(q_2) \qquad (8)$$

Then, let's take into consideration that:

the indices q_{12}, q_3, q_4 are proportional to degree of fulfillment of the corresponding conditions by the bridge;

the possible values of the indices q_{12}, q_3, q_4 are non-negative;

if any of the indices q_{12}, q_3 is equal to 0 then the quality is equal to 0 as well.

Thus, it is possible to write that $Q \sim q_T q_{12} q_3$. In this case if $Q > 0$ the bridge is probably correct (and the probability increases with increasing Q), otherwise it is unconditionally wrong.

Then, let's take into account, that condition 3.4 often is not fulfilled even if the bridge is absolutely correct. Nevertheless, fulfillment of this condition by the bridge increases its probability to be correct. Thus, let's express Q by the following formula:

$$Q = q_T q_{12} q_3 (1 + k_i q_4) \qquad (9)$$

where k_i is the *factor of interpolation* that allows controlling influence of condition 3.4 on the value of Q.

On the base of formula 9 by substitution of the formulas 2, 5, 7, 8 finally we obtain the following expression of Q:

$$Q = (\vec{n}_X - \vec{d}(\vec{n}_X \vec{d}))\vec{s}^{XO} q_{12}(1 + k_i q_4) \qquad (10)$$

4.2 Definition of the indicator field

As an elementary source of the indicator field let's define the aggregate of the following components: a point (O) on the boundary of an island, that isn't the base island; the external normal vector ($\vec{n}^O$) to the island surface at the point; the tangent vector ($\vec{\tau}^O$) at this point; a scalar value (h^O) that is the "charge" associated with the given source.

As an object, that the field affects let's define the aggregate of a point (X) and a unit vector ($\vec{n}^X$) at the point.

Let's assume that such source creates a field that makes influence on such object by force $\vec{f}$ acting at X and defined by the following equation:

$$\vec{f} = h^O \vec{d} Q \qquad (11)$$

where $\vec{d}$ is defined by formula 6; Q is the quality of bridge XO defined by formula 10.

Thus, point O "attracts" point X, if the bridge XO is probably topologically correct and "repulses" it otherwise. The attraction force is proportional to the quality of this bridge.

Definition 8. Let's define the tension of field H created by a given source $\{O, \vec{n}^O, \vec{\tau}^O, h^O\}$ at a given point (X) as an operator H, whose result of action on a unit vector ($\vec{n}$) at this point is the force acting at the point:

$$\vec{f}(\vec{n}) = \mathrm{H}\vec{n}$$

Let's also define the length of the force vector as the scalar value of the tension.

Like that, operator H is a function of one scalar and four vector arguments: $\mathrm{H} = \mathrm{H}(O, \vec{n}^O, \vec{\tau}^O, h^O, X)$. This operator should provide the property of superposition of field H. Therefore, for this operator for any space point X, any unit vector $\vec{n}$, and any value of "charge" h^O the following equations would be true:

$$\mathrm{H}_1 + \mathrm{H}_2 = \mathrm{H}_2 + \mathrm{H}_1$$

$$(\mathrm{H}_1 + \mathrm{H}_2) + \mathrm{H}_3 = \mathrm{H}_1 + (\mathrm{H}_2 + \mathrm{H}_3)$$

$$\mathrm{H}_1\vec{n} + \mathrm{H}_2\vec{n} = (\mathrm{H}_1 + \mathrm{H}_2)\vec{n}$$

$$\mathrm{H}(O, \vec{n}^O, \vec{\tau}^O, kh^O, X) = k\mathrm{H}(O, \vec{n}^O, \vec{\tau}^O, h^O, X)$$

where k is a constant.

Field H providing fulfillment of these conditions is a tensor field. The tension of this field created by the given elementary source $\{O, \vec{n}^O, \vec{\tau}^O, h^O\}$ at the given point X is represented by the following matrix:

$$\mathrm{H}(O, \vec{n}^O, \vec{\tau}^O, h^O, X) = h^O \mathrm{AB} \quad (12)$$

where

$$\mathrm{A} = g\begin{bmatrix} d_x s_x^{XO} & d_x s_y^{XO} & d_x s_z^{XO} \\ d_y s_x^{XO} & d_y s_y^{XO} & d_y s_z^{XO} \\ d_z s_x^{XO} & d_z s_y^{XO} & d_z s_z^{XO} \end{bmatrix} \quad (13)$$

where $\vec{d}$ is defined by formula 6, $\vec{s}^{XO}$ is defined in accordance with formula 1, $d_{x,y,z}$, $s_{x,y,z}^{XO}$ are the components of the corresponding vectors; $g = q_{12}(1 + k_i q_4)$;

$$\mathrm{B} = \begin{bmatrix} 1 - d_x^2 & -d_x d_y & -d_x d_z \\ -d_y d_x & 1 - d_y^2 & -d_y d_z \\ -d_z d_x & -d_z d_y & 1 - d_z^2 \end{bmatrix} \quad (14)$$

In addition let's note that to increase robustness of the method, formula 11 can be supplemented by the following condition of shielding:

$$\vec{f} = \delta^{XO} h^O \vec{d} Q \quad (15)$$

where $\delta^{XO} = 0$ if segment XO crosses A and $\delta^{XO} = 1$ otherwise.

4.3 Determination of the normal vectors

Force lines of defined above field H are defined only concerning some function $\vec{n} = \vec{n}(X)$ that associates a normal vector with each space point. During tracking a force line this function should be defined at each point of the iterative process. Let's take into account that at each of these points the plane allowed for the normal vector is known. For the origin point of the force line (it is a point on the boundary of the base island) it is the plane that is perpendicular to the tangent vector at the point. For each of others points it is the plane that is perpendicular to the tangent line to the force line at this point. Let's define, that at a given point of the iterative process the normal vector is the solution of the following system:

for the origin point:

$$\begin{cases} |\mathrm{H}\vec{n}| = \max \\ |\vec{n}| = 1 \\ \vec{n}\vec{\tau} = 0 \end{cases} \quad (16a)$$

for each of other points:

$$\begin{cases} |\mathrm{H}\vec{n}| = \max \\ |\vec{n}| = 1 \\ \vec{n}\vec{p} = 0 \end{cases} \quad (16b)$$

where $\vec{\tau}$ is the tangent vector at the point; $\vec{p}$ is the vector that is tangent to the trajectory of the force line at the given point.

5. RESULTS

The theoretical concepts described above have been verified by an experimental implementation. In this implementation as a "darning" method (introduced in definition 2) the simplest method has been chosen. We just calculate the point of the geometrical center of the boundary of a considered hole, and then we make a "triangle fan" inside the hole using the obtained central point. The choice of the given "darning" method means that for reduction an ICADM of class ICM3 (or ICM2) to an ICADM of class ICM1 we need to construct the greatest number of bridges in comparison with other more robust "darning" methods (like projection-based, spline-based, etc). This fact allows interpreting the obtained cost indices as the worst case for tested ICADMs.

The created implementation has been applied to repairing several ICADMs. The testing has been made for each of the two following modes of work of the algorithm: "light" mode (let's denote it $M1$) and "full" mode (let's denote it $M2$). The "light" mode doesn't use the shielding condition (defined by formula 15) and doesn't take into account the influence of free points on the length of a bridge (introduced in subsubsection 4.1.5).

Here the results for four tested ICADMs are adduced. The visual results are shown in the figures 7, 8, 9, and 10. For each one the figure with letter "a" shows the input ICADM, the figure with letter "b" shows the finally obtained CAD-model.

In figure 8 along with examples of successful application of the method to reconstruct surface inside complex holes a remaining defect is marked by circle. Note, that incompleteness of the given ICADM in the corresponding region doesn't allow considering this region as one having a practical value.

In figure 11 fragments of the total sets of bridges created for the given ICADMs are shown. In this figure one can see that force lines of the field in general have behavior that is similar to behavior of spline curves.

In figure 12 an example (a fragment of model "Woman-1") of influence of the factor of interpolation (k_i) introduced in subsubsection 4.1.8 is shown.

The numerical results of the tests are shown in table 1. In this table the following denotations are used: N_{bnd} is the total number of boundary points; N_{bdg} is the total number of iterative points of constructed bridges; for each processing mode the corresponding time (denoted by t) of the reconstruction is adduced; for processing in mode $M1$ there is an additional information field (denoted by "Art.") about appearing any artifacts in comparison with the result of processing in mode $M2$. These artifacts for the ICADMs „Bunny-2" and „Bunny-3" are shown in the figures 13 and 14 respectively.

These results have been obtained on a PC with a 500MHz Pentium-3 CPU.

ICADM	Class	N_{bnd}	N_{bdg}	$M1$		$M2$
				t,s	Art.	t,s
„Bunny-1"	ICM2	2424	749	11	No	30
„Bunny-2"	ICM2	2319	3615	16	Yes	66
„Woman-1"	ICM2	1526	7938	34	No	283
„Bunny-3"	ICM3	3272	4150	48	Yes	185

Table 1

In general, cost of the algorithm mainly depends on properties of input topology of boundaries and distribution of free points. Therefore behavior of the cost can't be expressed by a simple "cost equation". Very roughly the cost can be estimated by the following expression:

$$C \sim kR^{\alpha+1}L\left(\frac{S_{\overline{A}}}{S_{smp}}\right)^{\beta} \tag{17}$$

where R is the typical remoteness of a boundary from neighbor boundaries; L is the total length of the boundaries; $S_{\overline{A}}$ is the total area of $\overline{A}$; S_{smp} is the typical area of a simple hole; $\alpha \in [1,3]$ but in practice α doesn't essentially differ from 1, the part "$+1$" expresses cost of calculation of the shielding factor δ defined by formula 15; $\beta \in [0,\frac{1}{2}]$ with dependence on the topology; k is a constant.

A comparison with the warping-based methods (mentioned in section 2) has been made analytically, because the cost of solution of the used type of differential equations is known and information about the number of iterations is adduced in the corresponding works. The following two cases of the modeling have been considered: relatively simple topology, when only local modeling of the membrane (mesh) in regions corresponding to $\overline{A}$ is required; complicated case, when to obtain a proper result the modeling entire membrane is needed. The proposed method shows convincing theoretical advantage over the warping in the first case while working in mode $M1$ and in the second case while working in mode $M2$.

6. CONCLUSION

Thus, the presented method substantially meets the requirements formulated in section 2. As its disadvantages high cost and sensitivity to precision of floating point operations can be mentioned. At the same time the described method has very good capabilities for parallelization, owing to the fact that the processes of determination of the field tension at points are absolutely independent. And the processes of determination of contributions of considered field sources to the total field tension at a given point are independent as well.

7. REFERENCES

[1] Algorri M-E., Schmitt F. *Surface reconstruction from unstructured 3d data. Computer Graphics forum, 15(1), pp. 47-60, 1996.*

[2] Aldasoro C., Algorri M-E. *A combined algorithm for image segmentation using neural networks and 3D surface reconstruction using dynamic meshes. Revista Mexicana de Ingeniería Biomedica, XXI(3), pp. 73–81, 2000.*

[3] Vasilescu M., Terzopoulos D. *Adaptive Meshes and Shells: Irregular Triangulation, Discontinuities, and Hierarchical Subdivision. Proceedings of the IEEE Computer Vision and Pattern Recognition Conference (CVPR'92), Champaign, IL, June, 1992, pp. 829–832.*

[4] Emelyanov A. *Robust Surface Reconstruction Strategy for Large Clouds of Points. Technical Report DCSE/TR-2003-07, Department of Computer Science, University of West Bohemia, Plzen, Czech Republic, 2003.*

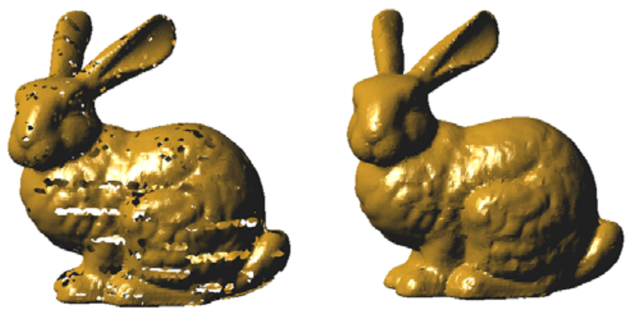
About the authors

Alexander Emelyanov is a Ph.D. student at University of West Bohemia, Department of Department of Computer Science. His contact email is aiem@kiv.zcu.cz.

Vaclav Skala is a professor at University of West Bohemia, Department of Department of Computer Science. His contact email is skala@kiv.zcu.cz.

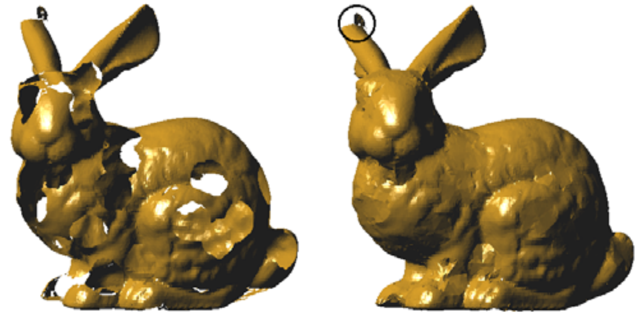
Acknowledgments

This work was supported by the Ministry of the Czech Republic – project MSM235200005 and by Microsoft Research Ltd. - project 2003-178



a b

Figure 7: "Bunny-1" (processed in mode $M1$)

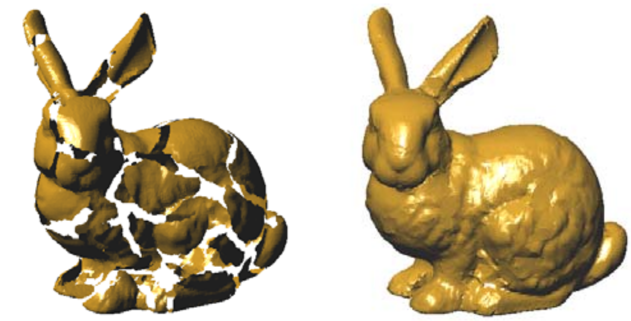


a b

Figure 8: "Bunny-2" (processed in mode $M2$)

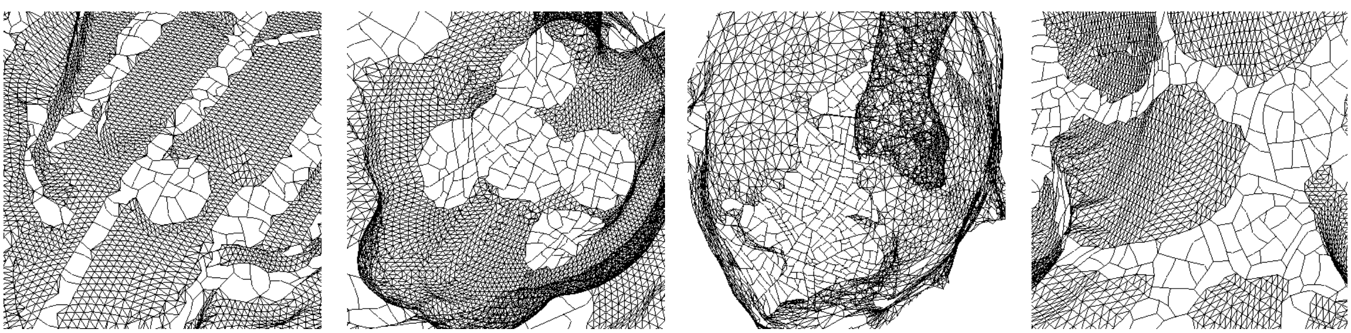
a b

Figure 9: "Woman-1" (processed in mode $M1$)



a b

Figure 10: "Bunny-3" (processed in mode $M2$)



a: "Bunny-1" **b:** "Bunny-2" **c:** "Woman1" **d:** "Bunny-3"

Figure 11: fragments of the sets of created bridges

a: $k_i = 0$ **b:** $k_i = 1$

Figure 12

Figure 13

Figure 14